

COURSE: AP Calculus AB	GRADE: 12
UNIT 1: Functions and Graphs	TIME FRAME: 7 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:	
M11.A.1	Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	Understand the meanings of operations, use operations and understand how they relate to each other.
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	Apply appropriate techniques, tools and formulas to determine measurements.
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	Demonstrate an understanding of patterns, relations and functions.
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.

M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.
M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	Analyze change in various contexts.
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:	UNIT OBJECTIVES:
<p>Numbers and Operations</p> <ul style="list-style-type: none"> Understand numbers, ways of representing numbers, relationships among numbers, and number systems. Understand meanings of operations and how they relate to one another. Compute fluently and make reasonable estimates. <p>Algebra</p> <ul style="list-style-type: none"> Understand patterns, relations, and functions. Represent and analyze mathematical situations and structures using algebraic symbols. Use mathematical models to represent and understand quantitative relationships. Analyze change in various contexts. <p>Geometry</p> <ul style="list-style-type: none"> Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. Use visualization, spatial reasoning, and geometric modeling to solve problems. <p>Measurement</p> <ul style="list-style-type: none"> Understand measurable attributes of objects and the units, systems, and processes of measurement. Apply appropriate techniques, tools, and formulas to determine measurements. <p>Problem Solving</p> <ul style="list-style-type: none"> Build new mathematical knowledge through problem solving. Solve problems that arise in mathematics and in other contexts. Apply and adapt a variety of appropriate strategies to solve 	<p>1.1 Use previously learned mathematical skills and concepts to analyze and algebraically confirm the facts and completeness of a graph that are determined technologically. Predict behavior that is hidden from view on the calculator screen.</p> <p>1.2 Use technology as a tool to investigate mathematical concepts and ideas to provide support for analytical work and to solve problems using approximation when analytic methods either fail or are impractical.</p> <p>1.3 Use previously learned information about relations, functions, their graphs and transformations, their inverses, and their properties and formulas.</p> <p>1.4 Local and global behavior of functions is considered numerically, analytically, and by utilizing the support of graphing technology.</p> <p>1.5 Model real-world phenomena.</p>

problems.

- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

<p>ACTIVITIES:</p> <p>Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize, and complete mathematical tasks.</p> <p>Seek assistance from the instructor or other facilitators whenever necessary.</p> <p>Use a graphing utility to draw graphs and determine a viewing window that gives the best complete graph.</p> <p>Sketch graphs analytically.</p> <p>Determine domain, range, intercepts, maximums, minimums, increasing and decreasing intervals, inflection points, and concavity of relations and functions.</p> <p>Use the vertical and horizontal line tests to determine whether a relation is a function and whether the inverse is a function.</p> <p>Find slope and equations for lines using various formulas.</p> <p>Use the definition of absolute value and its properties.</p> <p>Graph the absolute value function and other piecewise functions.</p> <p>Use the distance formula.</p> <p>Test analytically for symmetry.</p> <p>Test analytically to determine whether a function is even, odd, or neither.</p> <p>Graph the greatest integer function.</p> <p>Use the algebraic properties of functions.</p> <p>Use the geometric transformations of shifting, reflecting, stretching and shrinking to analytically graph variations of known graphs.</p> <p>Write equations for geometrically transformed graphs.</p> <p>Use the general equation of a circle.</p> <p>Determine inverse relations and inverse functions (compositional) graphically.</p> <p>Test for one-to-one.</p> <p>Determine inverses of linear, radical, rational, exponential and transcendental functions analytically.</p> <p>Use the properties of logarithms.</p>	<p>ASSESSMENTS:</p> <p>Homework Quizzes Tests Projects</p> <p>REMEDIATION:</p> <p><i>Precalculus Mathematics: A Graphing Approach</i>, 2nd ed., Demana et. al. Resource Manual</p> <p><i>Precalculus Mathematics: A Graphing Approach</i>, 2nd ed., Demana et. al. Graphing calculator Manual</p> <p>Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn.</p> <p>Lecture Guide and Student Notes</p> <p>Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005.</p> <p>Logarithm Combination Rules</p> <p>College Board Special Focus: Fundamental Theorem of Calculus. 2006.</p> <p>Describing Vectors (BC unit 9 only)</p> <p>Adding Vectors Graphically (BC unit 9 only)</p> <p>Vector Addition and Scalar Multiplication (BC unit 9 only)</p> <p>Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only)</p> <p>ENRICHMENT:</p> <p>AP Resources www.apcentral.collegeboard.com</p> <p>Master the AP Calculus AB and BC Test. Kelley, Michael. Thomson Learning. 2003</p> <p>Mathematics Calculus AB. Brook, Donald. REA. 1995</p> <p>Mathematics Calculus BC. Brook, Donald. REA. 1995</p> <p>Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998</p>
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<p>Graph exponentials and logarithmic functions.</p> <p>Use radian and degree measure and their conversions.</p> <p>Find arc length.</p> <p>Graph the trigonometric functions and variations and determine their respective domain, range, period, amplitude, and asymptotes (whenever applicable).</p> <p>Use the right triangle relationships.</p> <p>Graph the inverse trigonometric functions and variations.</p> <p>Use the trigonometric identities.</p> <p>Solve equations and inequalities.</p> <p>Solve various problems that model real-world Phenomena.</p>	<p>RESOURCES:</p> <p>Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy</p> <p>Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007.</p> <p>TI-84 Plus Instructional Manual</p>
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Revised 8/08

COURSE: AP Calculus AB	GRADE: 12
UNIT 2: Limits and Continuity	TIME FRAME: 10 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:	
M11.A.1	Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	Understand the meanings of operations, use operations and understand how they relate to each other.
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7 = x^{42}$) and powers of products $(2x^2)^3 = 8x^6$.
M11.B.2	Apply appropriate techniques, tools and formulas to determine measurements.
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	Demonstrate an understanding of patterns, relations and functions.
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.

M11.D.2.2	Simplify expressions involving polynomials.
M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	Analyze change in various contexts.
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:

Numbers and Operations

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
- Compute fluently and make reasonable estimates.

Algebra

- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Geometry

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement

- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

UNIT OBJECTIVES:

- 2.1 Intuitively understand the notion of limit. Use it to help explain and predict observed local and global behavior of a function; and, realize that the calculus is built on the idea (concept) of limit.
- 2.2 Intuitively understand that the central idea of continuity is correctives and use its definition to analytically confirm continuity at a point.
- 2.3 Model real-world phenomena.
- 2.4 Connect and use previously learned skills and concepts with new analytical methods to confirm data suggested by technology. Use technology to support all analytical and numerical facts.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

ACTIVITIES:

Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize and complete mathematical tasks.

Seek assistance from the instructor and other facilitators whenever necessary.

Discuss and construct graphs and algebraic functions that have and do not have limits.

Paraphrase and use the limit properties.

Analytically confirm observed local and end behavior of a computer graph and predict hidden behavior.

Determine one-sided and two-sided limits.

Discuss and construct graphs and algebraic functions that are and are not continuous.

Paraphrase and use the analytic test to determine continuity at a point.

Paraphrase the concept of local linearity and construct initial concept of tangent lines.

Use the algebraic properties of continuous functions and redefine functions that have a removable discontinuity.

Paraphrase and use the max-min-and Intermediate Value Theorems for continuous functions.

Use the sandwich theorem to verify $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ (key fact in the analytic derivation of derivatives of trigonometric functions).

Determine finite limits as ∞ .

Determine the nature of a graphs end behavior when $\lim_{x \rightarrow \infty} f(x)$ does not exist.

$x \rightarrow \infty$

Determine the nature of the graphs local behavior when $\lim_{x \rightarrow \infty} f(x)$ does not exist.

$x \rightarrow \infty$

Determine equations of vertical and horizontal asymptotes.

Model real-world phenomena of compound interest applications, natural behavior, and connectivity.

ASSESSMENTS:

- Observation and questioning
- Discussions
- Projects and investigations
- Mathematical writing
- Homework
- Quizzes and Tests

REMEDIATION:

Precalculus Mathematics: A Graphing Approach, 2nd ed., Demana et. al. Resource Manual

Precalculus Mathematics: A Graphing Approach, 2nd ed., Demana et. al. Graphing calculator Manual

Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn.

Lecture Guide and Student Notes

Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005.

Logarithm Combination Rules

College Board Special Focus: Fundamental Theorem of Calculus. 2006.

Describing Vectors (BC unit 9 only)

Adding Vectors Graphically (BC unit 9 only)

Vector Addition and Scalar Multiplication (BC unit 9 only)

Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only)

ENRICHMENT:

AP Resources

www.apcentral.collegeboard.com

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Mathematics Calculus AB. Brook, Donald. REA. 1995

Mathematics Calculus BC. Brook, Donald. REA. 1995

Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998

	<p>RESOURCES:</p> <p>Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy</p> <p>Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007.</p> <p>TI-84 Plus Instructional Manual</p>
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Revised 8/08

COURSE: AP Calculus AB	GRADE: 12
UNIT 3: Derivatives	TIME FRAME: 18 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:	
M11.A.1	Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	Understand the meanings of operations, use operations and understand how they relate to each other.
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	Apply appropriate techniques, tools and formulas to determine measurements.
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	Demonstrate an understanding of patterns, relations and functions.
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.

M11.D.2.2	Simplify expressions involving polynomials.
M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	Analyze change in various contexts.
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

<p>NCTM STANDARDS:</p> <p>Numbers and Operations</p> <ul style="list-style-type: none"> Understand numbers, ways of representing numbers, relationships among numbers, and number systems. Understand meanings of operations and how they relate to one another. Compute fluently and make reasonable estimates. <p>Algebra</p> <ul style="list-style-type: none"> Understand patterns, relations, and functions. Represent and analyze mathematical situations and structures using algebraic symbols. Use mathematical models to represent and understand quantitative relationships. Analyze change in various contexts. <p>Geometry</p> <ul style="list-style-type: none"> Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. Use visualization, spatial reasoning, and geometric modeling to solve problems. <p>Measurement</p> <ul style="list-style-type: none"> Understand measurable attributes of objects and the units, systems, and processes of measurement. Apply appropriate techniques, tools, and formulas to determine measurements. <p>Problem Solving</p> <ul style="list-style-type: none"> Build new mathematical knowledge through problem solving. Solve problems that arise in mathematics and in other contexts. Apply and adapt a variety of appropriate strategies to solve 	<p>UNIT OBJECTIVES:</p> <p>3.1 Interpret the notion of derivative geometrically. Use it to measure the rate at which things change; and, recognize that derivatives are substitution instances on limit functions.</p> <p>3.2 Use the basic rules to analytically find derivatives explicitly and implicitly.</p> <p>3.3 Understand and apply statements and theorems about derivatives (i.e. the Mean Value Theorem, L'Hôpital's Rule and the relationship between continuity and differentiability) to model and solve real-world problems.</p> <p>3.4 Use technology to support all analytic and numerical facts; and, connect and use previously learned skills and concepts with new analytical methods to confirm data suggested by technology.</p>
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problems.

- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
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- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
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Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

<p>ACTIVITIES: Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize and complete mathematical tasks.</p> <p>Seek help from the instructor and other facilitators whenever necessary.</p> <p>Model real world phenomena.</p> <p>Determine slopes of secant lines and tangent lines graphically and analytically and determine their relationship to the value of the derivative of a point.</p> <p>Use the definition of derivative to analytically find the derivative function as a limit (equivalent forms).</p> <p>Investigate one sided derivatives and differentiability of a function at a point.</p> <p>Use the graphing utility and symmetric difference form of derivative to calculate the value of the derivative at a point.</p> <p>Derive and apply the differentiation rules.</p> <p>Determine average and instantaneous velocity and acceleration and other rates of change.</p> <p>Determine the derivatives of composite functions via application of the Chain Rule.</p> <p>Use implicit differentiation to find derivations of the union of functions and fractional powers of differentiable functions.</p> <p>Determine and apply the relationships that exists between differentiability and continuity.</p> <p>Apply the Mean Value Theorem and Rolle's Theorem.</p> <p>Apply L`Hôpital`s Rule to determine limits.</p> <p>Use differential formulas to find linear approximations.</p> <p>Use derivative formulas for trigonometric functions.</p>	<p>ASSESSMENTS:</p> <ul style="list-style-type: none"> • Observation and questioning • Discussions • Projects and investigations • Mathematical writing • Homework • Quizzes and Tests <p>REMEDATION:</p> <p><i>Precalculus Mathematics: A Graphing Approach</i>, 2nd ed., Demana et. al. Resource Manual</p> <p><i>Precalculus Mathematics: A Graphing Approach</i>, 2nd ed., Demana et. al. Graphing calculator Manual</p> <p>Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn. Lecture Guide and Student Notes</p> <p>Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005.</p> <p>Logarithm Combination Rules</p> <p>College Board Special Focus: Fundamental Theorem of Calculus. 2006.</p> <p>Describing Vectors (BC unit 9 only)</p> <p>Adding Vectors Graphically (BC unit 9 only)</p> <p>Vector Addition and Scalar Multiplication (BC unit 9 only)</p> <p>Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only)</p> <p>ENRICHMENT:</p> <p>AP Resources www.apcentral.collegeboard.com</p> <p>Master the AP Calculus AB and BC Test. Kelley, Michael. Thomson Learning. 2003</p> <p>Mathematics Calculus AB. Brook, Donald. REA. 1995</p> <p>Mathematics Calculus BC. Brook, Donald. REA. 1995</p> <p>Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998</p>
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	<p>RESOURCES:</p> <p>Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy</p> <p>Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007.</p> <p>TI-84 Plus Instructional Manual</p>
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Revised 8/08

COURSE: AP Calculus AB		GRADE: 12
UNIT 4: Applications of Derivatives		TIME FRAME: 14 Days
PA ACADEMIC STANDARDS FOR MATHEMATICS:		
M11.A.1	Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.	
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).	
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.	
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).	
M11.A.1.1.3	Simplify square roots.	
M11.A.2	Understand the meanings of operations, use operations and understand how they relate to each other.	
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.	
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).	
M11.A.2.1.2	Solve problems using direct and inverse proportions.	
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.	
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.	
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).	
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.	
M11.B.2	Apply appropriate techniques, tools and formulas to determine measurements.	
M11.B.2.1	Use and/or compare measurements of angles.	
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).	
M11.C.1	Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.	
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.	
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.	
M11.D.1	Demonstrate an understanding of patterns, relations and functions.	
M11.D.1.1	Analyze and/or use patterns or relations.	
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.	
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.	
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).	
M11.D.2	Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.	
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.	
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.	
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.	
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).	
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.	
M11.D.2.1.5	Solve quadratic equations using factoring.	
M11.D.2.2	Simplify expressions involving polynomials.	
M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.	
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.	

M11.D.2.2.3 Simplify algebraic fractions.

M11.D.3 Analyze change in various contexts.

M11.D.3.1 Describe and/or determine change.

M11.D.3.1.1 Identify, describe and/or use constant or varying rates of change.

M11.D.3.1.2 Determine how a change in one variable relates to a change in a second variable

M11.D.3.2 Compute and/or use the slope of a line.

M11.D.3.2.1 Apply the formula for the slope of a line to solve problems.

M11.D.3.2.2 Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.

M11.D.3.2.3 Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:**Numbers and Operations**

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
- Compute fluently and make reasonable estimates.

Algebra

- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Geometry

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement

- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.

UNIT OBJECTIVES:

4.1 Use the derivative to analyze and confirm the completeness of a graph that is determined technologically. Predict behavior that is hidden from view or a computer graph.

4.2 Determine and use corresponding characteristics between the graph of a function and its derivatives and vice-versa.

4.3 Model real-world applications of optimization problems and rate-of-change problems.

4.4 Use technology to support all analytical and numerical facts and use analytical methods to confirm data suggested by technology.

- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

ACTIVITIES:

Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize, and complete mathematical tasks.

Seek help from the instructor and other facilitators whenever necessary.

Use the derivative to find the slope of a curve at a point, tangent and normal lines, critical points, local maximum and minimum values, inflection points and intervals on which the graph is rising, falling, concave up and concave down.

Use Newton's method to approximate zeros of a function plus determine differentials and linear approximations of a function. [OPTIONAL]

Predict behavior that is hidden from view or a computer graph and confirm the completeness of the graph.

Find vertical, horizontal and slant asymptotes.

Find extreme values of a function.

Use and apply the Mean Value Theorem to theoretical and real-world phenomena.

Model real-world applications of average and instantaneous rates of change.

Model real-world applications of velocity and acceleration in linear motion.

Model related rates-of-change applications.

ASSESSMENTS:

- Observation and questioning
- Discussions
- Projects and investigations
- Mathematical writing
- Homework
- Quizzes and Tests

REMEDICATION:

Precalculus Mathematics: A Graphing Approach, 2nd ed., Demana et. al. Resource Manual

Precalculus Mathematics: A Graphing Approach, 2nd ed., Demana et. al. Graphing calculator Manual

Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn.

Lecture Guide and Student Notes

Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005.

Logarithm Combination Rules

College Board Special Focus: Fundamental Theorem of Calculus. 2006.

Describing Vectors (BC unit 9 only)

Adding Vectors Graphically (BC unit 9 only)

Vector Addition and Scalar Multiplication

<p>Model real-world optimization problems.</p> <p>Determine corresponding characteristics between the graph of a function and its derivative and vise-versa.</p>	<p>(BC unit 9 only)</p> <p>Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only)</p> <p>ENRICHMENT:</p> <p>AP Resources www.apcentral.collegeboard.com</p> <p>Master the AP Calculus AB and BC Test. Kelley, Michael. Thomson Learning. 2003</p> <p>Mathematics Calculus AB. Brook, Donald. REA. 1995</p> <p>Mathematics Calculus BC. Brook, Donald. REA. 1995</p> <p>Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998</p> <p>RESOURCES:</p> <p>Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy</p> <p>Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007.</p> <p>TI-84 Plus Instructional Manual</p>
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COURSE: AP Calculus AB	GRADE: 12
UNIT 5: Integrals	TIME FRAME: 23 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:	
M11.A.1	Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	Understand the meanings of operations, use operations and understand how they relate to each other.
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7 = x^{42}$) and powers of products $(2x^2)^3 = 8x^6$.
M11.B.2	Apply appropriate techniques, tools and formulas to determine measurements.
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	Demonstrate an understanding of patterns, relations and functions.
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	Analyze change in various contexts.
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:	UNIT OBJECTIVES:
<p>Numbers and Operations</p> <ul style="list-style-type: none"> Understand numbers, ways of representing numbers, relationships among numbers, and number systems. Understand meanings of operations and how they relate to one another. Compute fluently and make reasonable estimates. <p>Algebra</p> <ul style="list-style-type: none"> Understand patterns, relations, and functions. Represent and analyze mathematical situations and structures using algebraic symbols Use mathematical models to represent and understand quantitative relationships. Analyze change in various contexts. <p>Geometry</p> <ul style="list-style-type: none"> Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. Use visualization, spatial reasoning, and geometric modeling to solve problems. <p>Measurement</p> <ul style="list-style-type: none"> Understand measurable attributes of objects and the units, systems, and processes of measurement. Apply appropriate techniques, tools, and formulas to determine measurements. <p>Problem Solving</p> <ul style="list-style-type: none"> Build new mathematical knowledge through problem solving. Solve problems that arise in mathematics and in other contexts. Apply and adapt a variety of appropriate strategies to solve problems. 	<p>5.1 Interpret antidifferentiation as the inverse operation of differentiation and use basic integration techniques (including formulas, substitution and integration by parts) to analytically determine antiderivatives (indefinite integrals).</p> <p>5.2 Geometrically interpret the definite integral as the area under the graph of a positive function and analytically interpret it as a limit of sums.</p> <p>5.3 Use the basic formulas and the Fundamental Theorems of Calculus to analytically evaluate definite integrals. Use numerical methods to approximate definite integrals when analytic methods fail or are impractical.</p> <p>5.4 Use current technology to support analytical and numerical findings and use analytical methods (including previously learned skills and concepts) to confirm data suggested by technology.</p>

- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

<p>ACTIVITIES:</p> <p>Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize and complete mathematical tasks.</p> <p>Seek help from the instructor and other facilitators whenever necessary.</p> <p>Use the formulas and rules for antiderivatives (indefinite integrals).</p> <p>Use the integration by substitution method.</p> <p>Use the formulas, rules, and methods to antidifferentiate polynomial, rational, radical, exponential, logarithmic, and trigonometric functions.</p> <p>Use the rectangle approximation methods (RAM) to estimate area under a curve.</p> <p>Use the algebraic rules and formulas for finite sums.</p> <p>Express the exact area under a curve as a limit of Riemann sums.</p> <p>Find the exact area under a curve using RAM Riemann sums.</p> <p>Express the definite integral as a limit of Riemann sums and vice-versa.</p> <p>Compute Riemann sums using left, right, and midpoint evaluation points.</p> <p>Use the symbols and terminology of integration.</p> <p>Use the definite integral and its properties to find the area under a curve analytically and technologically.</p> <p>Evaluate discontinuous integrable functions to find area under a curve.</p> <p>Interpret and use the definition of average value and Mean Value Theorem for definite integrals.</p> <p>Discuss, interpret and apply the connections between differential and integral calculus.</p> <p>Discuss interpret and use the Fundamental Theorem of Calculus, (Part 1 and Part 2) to evaluate definite integrals and represent a particular antiderivative.</p> <p>Use technology to graph functions of the form $\int_a^x f(t)dt$ (functions whose explicit forms are unknown).</p> <p>Evaluate definite integrals analytically and confirm technologically (and, if possible, vise-versa).</p>	<p>ASSESSMENTS:</p> <p>Homework Quizzes Tests Projects</p> <p>REMEDIATION:</p> <p><i>Precalculus Mathematics: A Graphing Approach</i>, 2nd ed., Demana et. al. Resource Manual</p> <p><i>Precalculus Mathematics: A Graphing Approach</i>, 2nd ed., Demana et. al. Graphing calculator Manual</p> <p>Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn.</p> <p>Lecture Guide and Student Notes</p> <p>Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005.</p> <p>Logarithm Combination Rules</p> <p>College Board Special Focus: Fundamental Theorem of Calculus. 2006.</p> <p>Describing Vectors (BC unit 9 only)</p> <p>Adding Vectors Graphically (BC unit 9 only)</p> <p>Vector Addition and Scalar Multiplication (BC unit 9 only)</p> <p>Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only)</p> <p>ENRICHMENT:</p> <p>AP Resources www.apcentral.collegeboard.com</p> <p>Master the AP Calculus AB and BC Test. Kelley, Michael. Thomson Learning. 2003</p> <p>Mathematics Calculus AB. Brook, Donald. REA. 1995</p> <p>Mathematics Calculus BC. Brook, Donald. REA. 1995</p> <p>Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998</p>
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Use the Trapezoidal Rule to approximate the area under a curve.

Use Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

Use the definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

RESOURCES:

Calculus: Graphical, Numerical, Algebraic
2007

Finney, Demana, Waits, Kennedy

Barton, Brunsting, Diehl, Hill, Tyler, Wilson
Pearson Education AP Test Prep Series: AP
Calculus (student workbook for Finney
text)

Pearson Education. Boston, Massachusetts,
2007.

TI-84 Plus Instructional Manual

COURSE: AP Calculus AB	GRADE: 12
UNIT 6: Applications of Definite and Indefinite Integrals	TIME FRAME: 18 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:	
M11.A.1	Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	Understand the meanings of operations, use operations and understand how they relate to each other.
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	Apply appropriate techniques, tools and formulas to determine measurements.
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	Demonstrate an understanding of patterns, relations and functions.
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	Analyze change in various contexts.
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

<p>NCTM STANDARDS:</p> <p>Numbers and Operations</p> <ul style="list-style-type: none"> Understand numbers, ways of representing numbers, relationships among numbers, and number systems. Understand meanings of operations and how they relate to one another. Compute fluently and make reasonable estimates. <p>Algebra</p> <ul style="list-style-type: none"> Understand patterns, relations, and functions. Represent and analyze mathematical situations and structures using algebraic symbols. Use mathematical models to represent and understand quantitative relationships. Analyze change in various contexts. <p>Geometry</p> <ul style="list-style-type: none"> Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. Use visualization, spatial reasoning, and geometric modeling to solve problems. <p>Measurement</p> <ul style="list-style-type: none"> Understand measurable attributes of objects and the units, systems, and processes of measurement. Apply appropriate techniques, tools, and formulas to determine measurements. <p>Problem Solving</p> <ul style="list-style-type: none"> Build new mathematical knowledge through problem solving. Solve problems that arise in mathematics and in other contexts. Apply and adapt a variety of appropriate strategies to solve problems. Monitor and reflect on the process of mathematical problem solving. 	<p>UNIT OBJECTIVES:</p> <p>6.1 Use antiderivatives to solve separable differential equations and model real-world phenomena involving distance, velocity and acceleration with initial conditions, and growth and decay.</p> <p>6.2 Find area under and between curves, volumes of solids of revolution, and volumes of solids with known cross sections.</p> <p>6.3 Model real-world phenomena involving area and volume.</p> <p>6.4 Use technology to support all analytical and numerical facts. Use analytic methods including previously learned skills and concepts to confirm (whenever possible) data suggested by technology.</p>
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Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

<p>ACTIVITIES:</p> <p>Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize and complete mathematical tasks.</p> <p>Seek help from the instructor and other facilitators whenever necessary.</p> <p>Solve separable differential equations and real world initial value applications of growth and decay.</p> <p>Find velocity given acceleration and distance given initial values for velocity and acceleration.</p> <p>Find the distance traveled by a particle along a line.</p> <p>Find solution curves.</p> <p>Model other real-world phenomena given initial conditions.</p> <p>Find area between curves by integrating with respect to x and also with respect to y.</p> <p>Determine volumes of solids of revolution using disks and washers.</p> <p>Determine volumes of solids of revolution using cylindrical shells.</p> <p>Determine volumes of solids of revolution generated by overlapping regions.</p> <p>Determine volume of solids with known cross sections using the slicing method.</p> <p>Determine volumes of solids that model real world phenomena.</p> <p>Find the average value of a function.</p>	<p>ASSESSMENTS:</p> <p>Homework Quizzes Tests Projects</p> <p>REMEDIATION:</p> <p><i>Precalculus Mathematics: A Graphing Approach</i>, 2nd ed., Demana et. al. Resource Manual</p> <p><i>Precalculus Mathematics: A Graphing Approach</i>, 2nd ed., Demana et. al. Graphing calculator Manual</p> <p>Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn.</p> <p>Lecture Guide and Student Notes</p> <p>Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005.</p> <p>Logarithm Combination Rules</p> <p>College Board Special Focus: Fundamental Theorem of Calculus. 2006.</p> <p>Describing Vectors (BC unit 9 only)</p> <p>Adding Vectors Graphically (BC unit 9 only)</p> <p>Vector Addition and Scalar Multiplication (BC unit 9 only)</p> <p>Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only)</p> <p>ENRICHMENT:</p> <p>AP Resources www.apcentral.collegeboard.com</p> <p>Master the AP Calculus AB and BC Test. Kelley, Michael. Thomson Learning. 2003</p> <p>Mathematics Calculus AB. Brook, Donald. REA. 1995</p> <p>Mathematics Calculus BC. Brook, Donald. REA. 1995</p> <p>Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998</p>
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	<p>RESOURCES:</p> <p>Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy</p> <p>Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007.</p> <p>TI-84 Plus Instructional Manual</p>
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Revised 8/08

AP Calculus AB Course Description

The AP Calculus AB course is a one semester class meeting for 90 days, 90 minutes per day. Students who enroll in AB generally go on to take BC in the second semester and take the AP Calculus BC Exam. Conceptual understanding is emphasized, and there is an essential balance in the course between technology and pencil and paper calculations. Each student has his or her own TI-84 plus calculator issued by the school to use for the duration of the course.

Through lecture, independent exploration, practice and collaboration, the students gain an in-depth understanding of the material and make valuable connections between and among topics. At the beginning of each class, the students work together on an introductory problem or activity, discussing ideas and approaches to reach a solution. Students are encouraged to ask questions and engage in class discussions. Homework is collected weekly and graded for completeness and correctness. Students are expected to complete out of class assignments and may collaborate on them at the beginning of the next class, allowing students the opportunity to communicate mathematics verbally on a daily basis.

Course Outline

- I. Unit 1: Functions and Graphs (Precalculus Review) 6 Days
 - a. Linear Functions
 - b. Exponential Functions
 - c. Logarithmic Functions
 - d. Inverse Functions
 - e. Parametric Curves
 - f. Trigonometric Functions

Teaching Notes:

The focus is on the analysis of graphs and characteristics of functions. Local and global behavior of functions is considered numerically, analytically, and by utilizing the support of graphing technology. Students are expected to make connections between the various representations. Also, the importance of considering all of the representations is emphasized. For example, students are expected to realize that graphical representation is not always sufficient to describe characteristics of functions (e.g. removable discontinuity). From the very beginning of the course, students are encouraged to formulate mathematically sound arguments and reasonable solutions in both verbal and written form.

- II. Unit 2: Limits and Continuity (10 Days)
 - a. Limits
 - i. Informal definition of Limits and Notation
 - ii. Properties of Limits
 - iii. Formal Definition of Limit
 - iv. Limits Involving Infinity
 - v. Asymptotic Behavior
 - b. Definition of Continuity
 - i. Properties of continuity
 - ii. Continuous extension
 - iii. Intermediate Value Theorem
 - c. Tangent Lines
 - i. Average Rate of Change
 - ii. Instantaneous Rate of Change
 - iii. Tangents and Normal lines

Teaching Notes:

Limits are introduced using a graphical, numerical and analytical approach. One-sided limits are also evaluated and explored. Early on, the concept of infinity is incorporated through end behavior and asymptotic behavior of graphs. A strong emphasis is placed on reasons why a limit fails to exist. Graphical exploration is followed by analytical interpretation. Students use the Squeeze (Sandwich) Theorem to solidify their understanding of limits and begin their work with proofs. Verbal or written explanation is required along with their proof.

Continuity is initially approached from a graphical perspective. The idea of proof is revisited with the Intermediate Value Theorem. Through the understanding of local linearity and limits, students construct the concept of the tangent line.

- III. Derivatives (14 Days)
 - a. Definition of Derivative
 - b. Differentiability
 - c. Introduction to the relationship between the graphs of f and f'
 - d. Differentiation Rules
 - e. Rates of Change
 - f. Derivatives of Trigonometric Functions
 - g. Chain Rule
 - h. Parametric Differentiation
 - i. Implicit Differentiation
 - j. Derivatives of Exponential and Logarithmic Functions
 - k. Derivatives of Inverse Trigonometric Functions

Teaching Notes:

In the beginning of the unit, major emphasis is placed on the relationship between differentiability and continuity. Connections are made between the derivative, tangent line, slope of curve, average rate of change, and instantaneous rate of change. A major emphasis is placed on local linearity, requiring students to discuss the concept in their own words. Graphical exploration is utilized to "zoom-in" on various graphs to determine differentiability. Once conceptual understanding has been accomplished, the focus is shifted to computing derivatives and rules and techniques for differentiation. The differentiation rules are initially explored through the use of graphing technology. Students make conjectures about derivatives of polynomial, trigonometric and exponential functions. After exploration, many of the derivatives of trigonometric, exponential, logarithmic and inverse trigonometric functions are derived by the students using proofs. In particular, proofs of inverse trigonometric functions are written and verbally presented to the class by student teams.

- IV. Application of the Derivative (15 days)
 - a. Mean and Extreme Value Theorem
 - b. Increasing and Decreasing intervals
 - c. Concavity
 - d. Connecting f with f' and f''
 - e. Modeling and Optimization
 - f. Linearization
 - g. Differentials
 - h. Related Rates

Teaching Notes:

This unit is built on two major concepts. In the beginning of the unit, the emphasis is for students to make connections between a function and its first and second derivatives. This is done graphically, algebraically, and numerically. Also, the concept of the derivative as a rate of change is reinforced with reference to position, velocity and acceleration.

The second part of the unit focuses on the application of the derivative to solving problems, such as optimization and related rates problems. Students solve problems analytically, supporting their conclusions with the use of graphing technology. Students gain an appreciation for the usefulness of calculus through real-world scenarios. Also, the concept of approximating functions using tangent lines and differentials is explored graphically, numerically and analytically.

- V. Integrals (14 Days)
 - a. Riemann Sums
 - i. Right
 - ii. Left
 - iii. Midpoint
 - b. Definite Integrals
 - c. Fundamental Theorem of Calculus
 - d. Trapezoidal Approximation of Definite Integrals
 - e. Indefinite Integrals
 - f. Integration Techniques
 - i. Integration by Substitution
 - ii. Integration by Parts and Tabular Integration

Teaching Notes:

During this unit, there is a strong emphasis of the concept of a definite integral as a numerical value. Approximations involving Riemann Sums and Trapezoids are explored and compared. Right, Left, and Midpoint Riemann Sums and Trapezoidal approximations are done by hand and using programs for the graphing calculator. Students look at indefinite integrals as mathematical objects, mainly general antiderivatives. Integration properties, formulas and techniques are introduced and practiced. Relationships between functions and their antiderivatives are explored using graphing technology, particularly the relationship between position, velocity, and acceleration. One of the major concepts of this unit is the Fundamental Theorem of Calculus, Parts 1 and 2. Students are exposed to a variety of problems involving the FTC and its applications. Students are given excerpts from the College Board's AP Calculus 2005-2006 Professional Development Workshop Materials Special Focus: The Fundamental Theorem of Calculus. Released AB Free Response questions are used to facilitate proper written explanation of mathematical concepts.

- VI. Applications of Definite and Indefinite Integrals (22 days)
 - a. Differential Equations
 - b. Slope Fields
 - c. Separable Differential Equations
 - d. Law of Exponential Change
 - e. Euler's Method*
 - f. Partial Fraction Decomposition*
 - g. Logistic Differential equations*
 - h. Integrals as Net Change
 - i. Areas in the Plane
 - i. Area under curve
 - ii. Area between curves
 - j. Lengths of Curves*
 - k. Volumes
 - i. Disc method
 - ii. Washer method
 - iii. Shell method

Teaching Notes:

In the first part of this unit, the focus is on differential equations, using slope fields as a method to find solution curves and Euler's method as a numerical approximation of the function's value. Students draw slope fields by hand and use a slope field program for the graphing calculators. Discussion is facilitated by use of the overhead projection capabilities of the graphing calculator. Calculations for Euler's method are done by hand using step-by-step iteration and using a calculator program. Students develop written and verbal arguments for whether the solution produced by Euler's method is an over- or under-estimate of the actual solution based on both analytical and graphical analysis. After students have obtained a solid understanding of the concept of a differential equation, separable differential equations are introduced and exponential growth and decay are emphasized.

The second part of the unit focuses on applications of integrals as an accumulation of a rate of change, distance, area and volume. As an introduction to volume, computer-based solids of revolutions are explored at http://college.hmco.com/mathematics/larson/calculus_analytic/7e/students/3dgraphs/ch06.html. Volume is calculated using the disc, washer and shell methods, and using a variety of known cross-sectional areas.

* Indicates BC Topic covered in the first semester

Assessments

Exam: Chapter 1	Quiz: Definite Integrals
Quiz: Section 2.1, 2.2	Exam: Chapter 5
Exam: Chapter 2	Quiz: Section 6.2, 6.3
Quiz: Sections 3.5 – 3.9	Quiz: Section 6.1, 6.4
Exam: Chapter 3	Exam: Chapter 6
Quiz: Section 4.1 – 4.4	Quiz: Section 7.1, 7.2, 7.4
Quiz: Section 4.5, 4.6	Exam: Chapter 7
Exam: Chapter 4	Final Exam

In addition to teacher-generated quizzes and exams, students are also exposed to released Free-response questions from past AB exams and multiple choice practice problems written for AP Exam preparation. These problems are chosen and/or written to facilitate written student explanation of concepts.

Primary textbook

Finney, Demana, Waits, Kennedy
Calculus: Graphical, Numerical, Algebraic (Third Edition) AP Edition
Pearson Prentice Hall. Boston, Massachusetts, 2007.

Supplemental Resources

Textbooks:

Barton, Brunsting, Diehl, Hill, Tyler, Wilson
Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text)
Pearson Education. Boston, Massachusetts, 2007.

Larson, Hostetler, Edwards
Calculus (Sixth edition)
Houghton Mifflin Company. Boston, Massachusetts, 1998.

Websites:

1. AP Central

<http://apcentral.collegeboard.com/apc/Controller.jsp>

2. Houghton Mifflin: Calculus with Analytic Geometry

http://college.hmco.com/mathematics/larson/calculus_analytic/7e/students/3dgraphs/ch06.html.